

Chapter 2 Review

1-D Motion: Kinematics

• FRAME OF REFERENCE

Always keep in mind what our frame of reference is in a problem.
Typically it is "with respect to the Earth."

Book example:

Person walking on train.

Train $\rightarrow$ 80 km/h

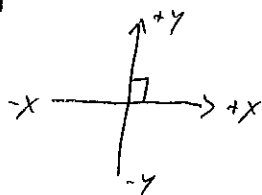
Person $\rightarrow$ 5 km/h

Ask: How fast is the person moving to an outside observer?

85 km/h.

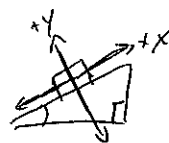
• Coordinate Axes

Always identify your coordinate axes when drawing or describing a problem!



Always 90° angle (perpendicular)

Sometimes you might need to draw it at an angle.

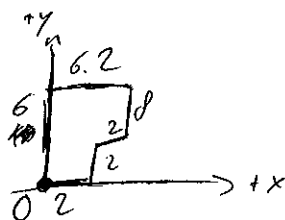


• Distance vs. Displacement

- Distance - total amount traveled.

- Displacement - distance from original point. Magnitude + Direction.

Example: Marathons:



Distance? 26.2 miles

Displacement? 0 miles.

Displacement is a vector (chapter 3)

Displacement:

Can be represented by ΔX

$$\Delta X = X_2 - X_1$$

" Δ " means "the change in"

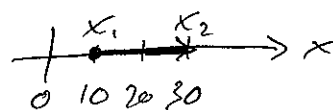
or

$$\Delta X = X_f - X_i$$

or

$$\Delta X = X - X_0$$

Example: $X_1 = 10 \text{ m}$ $X_2 = 30 \text{ m}$



$\Delta X = ?$ What direction? (+x)

$X_1 = 30 \text{ m}$ $X_2 = 10 \text{ m}$

$\Delta X = ?$ What direction? (-x)

Speed vs. Velocity

Ask: What's the difference?

Speed: ~~magnitude~~ how fast something is moving. (magnitude)

Velocity: how fast and in what direction
 @ Something is moving (magnitude + direction)

$$\text{Speed} = \frac{\text{Distance}}{\text{time}} \Rightarrow \text{average speed} = \frac{\text{total distance traveled}}{\text{total time elapsed}}$$

$$\text{velocity} = \frac{\text{displacement}}{\text{time}} \Rightarrow \text{average Velocity} = \frac{\text{total displacement } (\Delta X)}{\text{total time elapsed}}$$

We usually write as

$$\boxed{\bar{V} = \frac{\Delta X}{\Delta t}}$$

Acceleration

Change in velocity over time

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time elapsed}}$$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Note on instantaneous velocity + acceleration:

$\bar{v}$: average velocity v : instantaneous velocity
(velocity at a specific moment in time)

$\bar{a}$: average acceleration a : instantaneous acceleration.

Kinematic equations typically use "instantaneous" values.

EXAMPLES:

Velocity example: Problem 5:

A rolling ball moves from $x_1 = 3.4 \text{ cm}$ to $x_2 = -4.2 \text{ cm}$ during time from $t_1 = 3.0 \text{ s}$ to $t_2 = 6.1 \text{ s}$. What is average velocity? (Ask them where to start.)

$$\bar{v} = ?$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = -2.45161 \frac{\text{cm}}{\text{s}} = \boxed{-2.5 \frac{\text{cm}}{\text{s}}}$$

~~At this velocity, how long will it now take the ball to go from 4.2 cm to 3.0 cm?~~

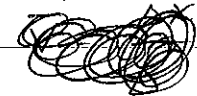
Examples Continued... At what time will the ball reach -30cm? (4)

What is x_1 here? (-4.2cm)

What is x_2 here? (-30cm)

What is t_1 here? (6.1s)

What is t_2 here? (?)



$$\frac{1}{V} \cdot \bar{V} = \frac{\Delta x}{\Delta t} \cdot \frac{1}{V}$$

$$\Rightarrow \Delta t = \frac{\Delta x}{\bar{V} \cdot \Delta t} \cdot \Delta t$$

$$\Rightarrow \Delta t = \frac{\Delta x}{\bar{V}}$$

$$\Rightarrow t_2 - t_1 = \left[\frac{(x_2 - x_1)}{\bar{V}} \right] + t_1$$

$$t_2 = \left[\frac{(x_2 - x_1)}{\bar{V}} \right] + t_1$$

$$\boxed{t_2 = 16.42 \text{ s}}$$

Example: Problem 10P: At highway speeds, a sports car can accelerate at a rate of 1.6 m/s^2 . How long does it take to accelerate from 80 km/h to 110 km/h.

What do we know?

$a, v_2, v_1, (t_1 = 0)$

Convert: $1.6 \frac{\text{m}}{\text{s}^2} \cdot \frac{1 \text{ km}}{1000 \text{ m}} = .0016 \text{ km/s}^2 \cdot \left(\frac{60 \text{ s}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ h}} \right)^2 =$

$$\bar{a} = \frac{\Delta v}{\Delta t} \Rightarrow \Delta t = \frac{\Delta v}{\bar{a}} = t_2 - t_1 = \frac{v_2 - v_1}{\bar{a}} = \frac{110 - 80}{5.2} = \boxed{5.2 \text{ sec}}$$

Conceptual Questions:

5

Q: ~~Q1, Q2, Q3, Q6, Q7, Q11~~ #1, #2, #3, #6, ~~Q7, Q11~~ #7, #11

If velocity of object is zero, does it mean acceleration is zero?

If acceleration is zero does it mean velocity is zero?

More Problems? Questions? and Q #9.

Motion at Constant Acceleration

$$1.) \quad \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Note: $\bar{a} = a$ if a is constant.

We will call $v_1 = v_0$, which is velocity at time = 0.

So: $t_1 = 0$; $t_2 = t$; $v_2 = v$

$$\Rightarrow a = \frac{v - v_0}{t - 0} = \frac{v - v_0}{t}$$

$$a \cdot t = v - v_0 \Rightarrow \boxed{v = v_0 + at}$$

2.) Remember how to find an average value?

$\bar{v}$ of a constantly accelerating object will just be:

$$\bar{v} = \frac{v_0 + v}{2}$$

and! $\bar{v} = \frac{\Delta x}{\Delta t} \Rightarrow \boxed{\bar{v} = \frac{x - x_0}{t}}$

Combine:

$$\frac{v_0 + v}{2} = \frac{x - x_0}{t}$$

16

Motion const. acc.

$$\frac{V_0 + V}{2} = \frac{x - x_0}{t} \Rightarrow \frac{1}{2} V_0 t + \frac{1}{2} V t = x - x_0$$

but: $V = V_0 + at$ $\nearrow$

$$\frac{1}{2} V_0 t + \frac{1}{2} V_0 t + \frac{1}{2} at^2 = x - x_0$$

$$\boxed{x = V_0 t + \frac{1}{2} at^2 + x_0}$$

3.) ~~$V = V_0 + at \Rightarrow V - V_0 = at$~~ ~~$\frac{V - V_0}{a} = t$~~

~~$x = x_0 + V_0 \left(\frac{V - V_0}{a} \right) + \frac{1}{2} a \left(\frac{V - V_0}{a} \right)^2$~~

$$x = x_0 + \bar{V} t \quad ; \quad \bar{V} = \frac{V_0 + V}{2}$$

$$x = x_0 + \left(\frac{V_0 + V}{2} \right) t$$

$$V = V_0 + at \Rightarrow t = \frac{V - V_0}{a} \nearrow$$

$$x = x_0 + \left(\frac{V_0 + V}{2} \right) \left(\frac{V - V_0}{a} \right)$$

$$x = x_0 + \frac{V^2 - V_0^2}{2a}$$

$$\boxed{V^2 = V_0^2 + 2a(x - x_0)}$$

THESE ARE THE KINEMATIC EQUATIONS!

Falling Objects

Galileo discovered that for free fall, all objects at a given location will fall ~~at the same~~ with the same constant acceleration. (if we neglect air resistance)

Acceleration due to gravity: $g = 9.80 \text{ m/s}^2$ (toward center of Earth)

This is fortunate. We can now solve free fall problems with the kinematic equations from earlier.

$$a \rightarrow g \quad x \rightarrow y$$

Example: Ball thrown upwards. (p. 33)

Ball is thrown upward with an initial velocity of 15.0 m/s

a) How high does it go? b) How long is it in the air?

a.) What do we know?

$$V_0 = 15.0 \frac{\text{m}}{\text{s}} \quad y_0 = 0 \quad a = -9.80 \text{ m/s}^2 \text{ (which direction?)}$$

$$V = ? \quad y = ? \quad t = ?$$

I pick 0

Which equation? $V^2 = V_0^2 + 2a(y - y_0)$

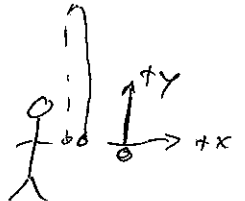
~~$V^2 = V_0^2 + 2a(y - y_0)$~~

~~$V^2 = V_0^2 + 2a(y - y_0)$~~

~~$V^2 = V_0^2 + 2a(y - y_0)$~~

$$V^2 = V_0^2 + 2a(y - y_0) \Rightarrow 0 = V_0^2 + 2ay$$

$$-V_0^2 = 2ay \Rightarrow y = \frac{-V_0^2}{2a} = \boxed{11.5 \text{ m}}$$



~~$V = V_0 + at$~~

Chapter 3.

Vectors (Displacement, acceleration, velocity)

A QUANTITY WITH MAGNITUDE + DIRECTION.

Scalar

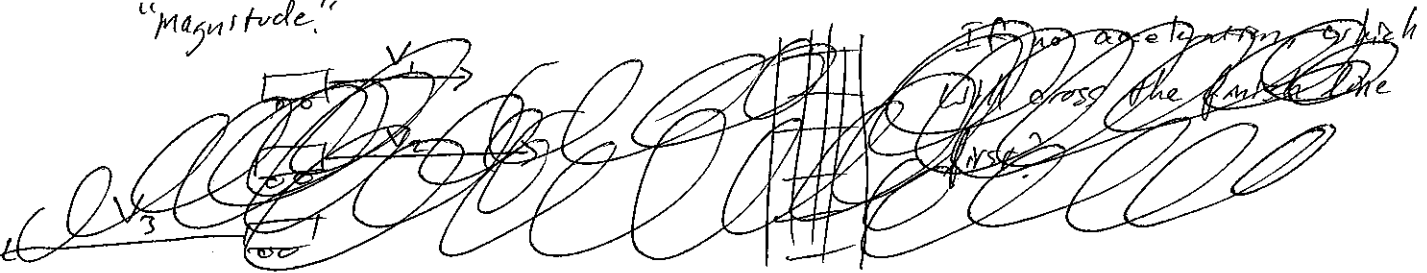
A QUANTITY with only a Magnitude.

~~mass, temperature, time, speed, distance~~

(mass, temperature, time, speed, distance)

Diagrams with Vectors.

When we draw a diagram of a problem involving vectors, we use arrows. Direction of arrow indicates "direction," size of arrow indicates "magnitude."



~~See~~ ptc. 3:1 on pg. 46: ~~At which point B the car starts? fastest?~~

From now on: Vector will be written in bold, or with arrow above it.

Velocity: $\vec{v}$ (or bold)

magnitude of velocity (speed): v (or italics)

Vector Addition

Because vectors are more than just a number, we have a special way of adding them, in order to preserve direction.

Graphically:

$\vec{D}$ displacement vector

$\vec{V}$ velocity vector

Tail-to-tip method

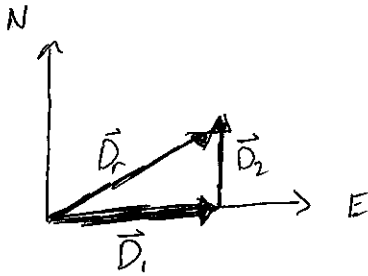
Ex: Fig. 3-3
A car drives east 10 ~~km~~ km, and north 5 km.

$$\vec{D}_1 = 10 \text{ km East}; \quad \vec{D}_2 = 5 \text{ km North.}$$

If we add these, we get a resultant vector:

$$\vec{D}_R = \vec{D}_1 + \vec{D}_2$$

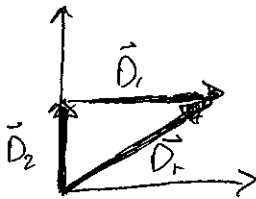
Note: ~~$D_R \neq D_1 + D_2$~~ $D_R \neq D_1 + D_2$ (magnitude)



Note: tail to tip!

Resultant: Tail of first to tip of last.

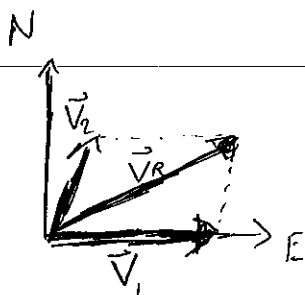
Just like normal numbers: $\vec{D}_1 + \vec{D}_2 = \vec{D}_2 + \vec{D}_1$?



Parallelogram Method

Boat: Water pushing to the East at $\vec{V}_1$.

Boat drives due North-North-East at $\vec{V}_2$.



$$\vec{V}_R = \vec{V}_1 + \vec{V}_2 = \vec{V}_2 + \vec{V}_1$$

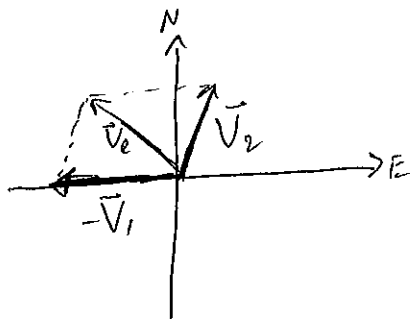
$\vec{V}_R$ is the actual motion of the boat!

Note: Not connecting tips!

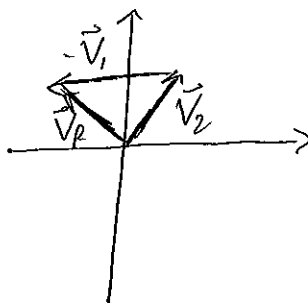
Conceptual Example 3-1: p. 48. (Check time!)

Subtraction of Vectors:

$$\vec{V}_2 - \vec{V}_1 = \vec{V}_2 + (-\vec{V}_1)$$



or



Multiplying by scalar:

$$\vec{V} = 1.5 \text{ m/s NNE} \quad \nearrow \vec{V}$$

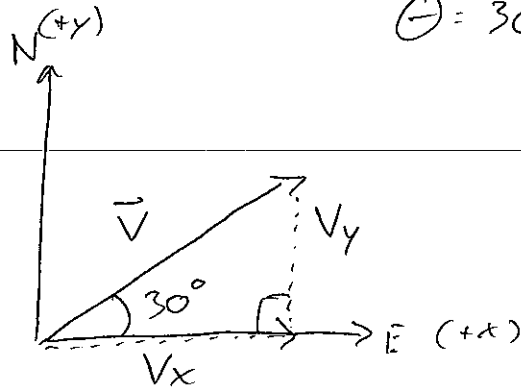
$$2 \cdot \vec{V} = 3.0 \text{ m/s NNE} \quad \nearrow 2\vec{V}$$

$$-2 \cdot \vec{V} = -3.0 \text{ m/s NNE or } 3.0 \text{ m/s SSW} \quad \searrow -2\vec{V}$$

Adding by Components.

14

~~Object~~ Object is moving with velocity $\vec{V}$ 30° north of east.
 $\Theta = 30^\circ$



We break this into components:

V_x and V_y

Note: Right triangle!

Recall:

Pythagorean theorem:

$$\boxed{V_x^2 + V_y^2 = V^2} \quad (\text{magnitude})$$

Trigonometry:

SOH CAH TOA

$$\sin \Theta = \frac{V_y}{V} \Rightarrow \boxed{V_y = V \sin \Theta}$$

$$\cos \Theta = \frac{V_x}{V} \Rightarrow \boxed{V_x = V \cos \Theta}$$

$$\boxed{\tan \Theta = \frac{V_y}{V_x}} \quad \text{or} \quad \boxed{\Theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)} \quad \text{in calculator.}$$

Now: If we were adding two vectors

$$\vec{V} = \vec{V}_1 + \vec{V}_2 \quad - \text{Break into components}$$

$$\Rightarrow V_x = V_{1x} + V_{2x}$$

Add the x-components ~~(V1cosΘ1 + V2cosΘ2)~~

$$\Rightarrow V_y = V_{1y} + V_{2y}$$

Add y-components
($V_1 \sin \Theta_1 + V_2 \sin \Theta_2$)

$$V^2 = V_x^2 + V_y^2$$

$$\text{And } \Theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$$

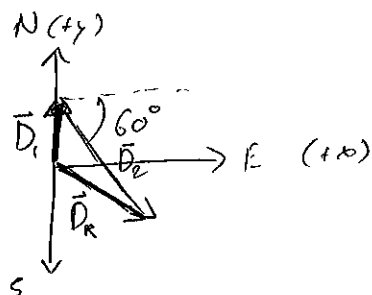
Note: $\Theta \neq \Theta_2 + \Theta_1$

Example:

Mail carrier 22.0 km N.

Drives 47.0 km ~~South~~ 60° South of East.

What is her displacement?



$$\vec{D}_R = \vec{D}_1 + \vec{D}_2$$

$$D_x = D_{1x} + D_{2x}$$

$$D_{1x} = 0 \text{ (vertical)} \quad \theta_1 = 90^\circ \quad \cos \theta_1 = 0$$

$$D_{2x} = \cancel{47.0 \text{ km}} D_2 \cos \theta = 23.5 \text{ km}$$

Note:

Calculators:

radians/degrees

$$D_{1y} = 22 \text{ km}$$

$$D_{2y} = -D_2 \sin 60$$

$$= -40.7 \text{ km}$$

(Note: Negative sign because angle is below horizontal.)

$$D_x = 23.5 \text{ km}$$

$$D_y = -18.7 \text{ km}$$

$$D = \sqrt{D_x^2 + D_y^2} = 30.0 \text{ km}$$

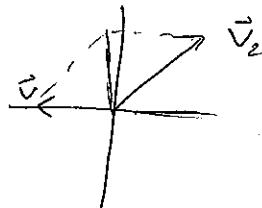
$$\tan \theta = \frac{D_y}{D_x} = -0.796$$

$$\theta = -38.5^\circ$$

Need Magnitude
+
Direction.

8.] $\vec{V}_1 = 6.6$ @ 0° from $-x$

$\vec{V}_2 = 8.5$ @ 45° above $+x$



a.) $V_{x1} = V_1 \cos \theta_1 = -6.6$ $V_{x2} = V_2 \cos \theta_2 = 6.01$

$V_{y1} = V_1 \sin \theta_1 = 0$ $V_{y2} = V_2 \sin \theta_2 = 6.01$

b.) $\vec{V} = \vec{V}_1 + \vec{V}_2$

$V_x = V_{x1} + V_{x2} = -.59$

$V_y = V_{y1} + V_{y2} = 6.01$

$V = \sqrt{V_x^2 + V_y^2} = \boxed{5.98}$

$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = -84.39^\circ$ from where? Draw Diagram. Tangent is tricky.

~~Note: $\theta = -84.39^\circ$~~
~~It is hard to measure & define your angles from the~~
~~clock point (usually 0° or 90°).~~

~~Here $\theta = 84.39^\circ$~~

~~84.39°~~

31.] A runner hopes to complete 10,000 m run in less than 30 min.

After 27 min, there are still 1100 m to go. For how long must runner accelerate at $.20 \text{ m/s}^2$ to achieve goal?

$$X_0 = 10,000_m - 1,100_m = 8900_m$$

$$X = 10000_m$$

$$V_0 = ? \quad \left(\frac{8900_m}{27 \text{ min}} = 5.4389 \text{ m/s} \right)$$

$$V = ? \left(\frac{1100_m}{3 \text{ min}} \right) = 6.1111 \text{ m/s}$$

$$a = .20 \text{ m/s}^2$$

$$V = V_0 + at \Rightarrow \boxed{t = \frac{V - V_0}{a} = 3.1 \text{ s}}$$

15.] Bowling ball traveling with constant speed hits the pins at the end of a lane 16.5 m long. Bowler hears the crash 2.50 s after he releases the ball. If $V_{\text{sound}} = 340 \text{ m/s}$, what is the speed of the ball?

$$t_1 = ?$$

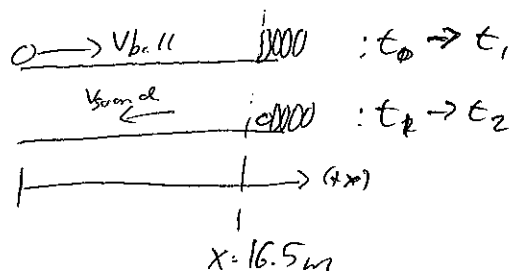
$$t_2 = 2.50 \text{ s}$$

$$t_0 = 0_s$$

$$V_{\text{ball}} = ?$$

$$V_{\text{sound}} = 340 \text{ m/s}$$

$$X = 16.5 \text{ m}$$



$$V_{\text{ball}} = \frac{X - X_0}{t_1 - t_0} = \frac{X}{t_1}$$

$$V_s = \frac{X_0 - X}{t_2 - t_1} \Rightarrow t_2 - t_1 = \frac{-X}{V_s}$$

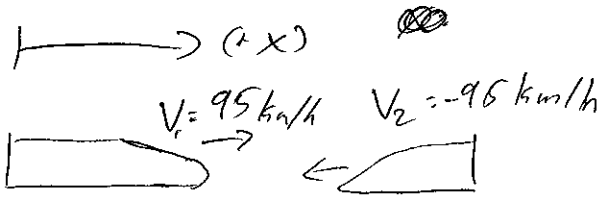
$$\Rightarrow t_1 = t_2 + \frac{X}{V_s} = 2.45 \text{ s}$$

$$V_{\text{ball}} = \frac{X}{t_1} \Rightarrow \boxed{V_{\text{ball}} = 6.73 \text{ m/s}}$$

11, 28, 37, 15, 31, 47

Q: 12, 13, 17

11.)



Change reference frame: to frame of Tram 2.

$$V'_1 = V_1 - V_2 = 95 - 95 = 190 \text{ km/h}$$

$$V'_2 = V_2 - V_2 = 0 \text{ km/h}$$

$$x_0 = 0$$

$$x_f = 0.5 \text{ km}$$

$$t = ?$$

$$a = 0 \text{ km/h}^2$$

$$\bar{v} = \frac{x - x_0}{t}$$

$$\Rightarrow \boxed{t = \frac{x - x_0}{\bar{v}} = .045 \text{ h} = 161.05 \text{ s}}$$

2.7 min

Chapter 3: Questions: 4, 5, 6.
7, 8, 2.

Chapter 3: Problems
6, 7, 8

Remember! $V_x = V \cos \theta$ $V_y = V \sin \theta$
 $V = \sqrt{V_x^2 + V_y^2}$ $\tan \theta = \frac{V_y}{V_x}$

$$\begin{aligned}\vec{V} &= \vec{V}_1 + \vec{V}_2 \\ V_x &= V_{x1} + V_{x2} \\ V_y &= V_{y1} + V_{y2} \\ (V_z &= V_{z1} + V_{z2})\end{aligned}$$

6.) $\vec{V} = \vec{V}_1 + \vec{V}_2$

$$V_x = V_{x1} + V_{x2} = 8.0 + 3.9 = 11.9$$

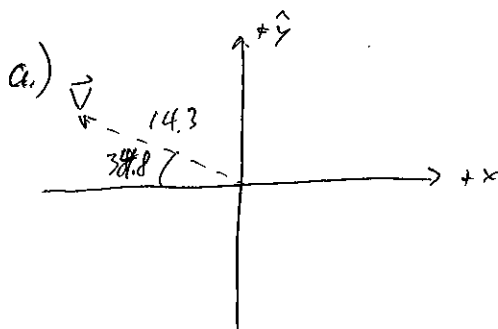
$$V_y = V_{y1} + V_{y2} = -3.7 + (-8.1) = -11.8$$

$$V_z = V_{z1} + V_{z2} = 0.0 + (-4.4) = -4.4$$

$$V^2 = V_x^2 + V_y^2 + V_z^2$$

$$V = \sqrt{V_x^2 + V_y^2 + V_z^2} = 17.3$$

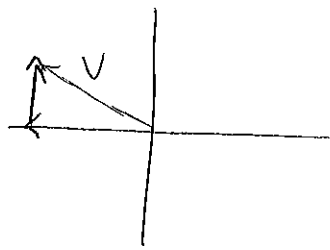
7.) a.) $V = 14.3$
 $\theta = 34.8^\circ$ above $-x$



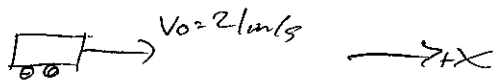
b) $V_y = V \sin \theta = 8.16$
 $V_x = V \cos \theta = 11.7$

c.) $V = \sqrt{V_x^2 + V_y^2} = 14.2645$

$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = 34.8933$$



26.)



$$V_0 = 21.0 \text{ m/s}$$

$$V = 0 \text{ m/s}$$

$$t = 6.00 \text{ s}$$

$$x_0 = 0 \text{ s}$$

$$x = ?$$

$$a = \frac{V - V_0}{t} = -3.5 \text{ s}$$

$$x = x_0 + V_0 t + \frac{1}{2} a t^2$$

$$x = 63.0 \text{ m}$$

37.)

$$t = 3.0 \text{ s}$$

$$a = -9.8 \text{ m/s}^2$$

$$V_0 = ?$$

$$V = ? = -V_0$$

$$\left. \begin{array}{l} y_0 = 0 \\ y = 0 \end{array} \right\} \text{ so } V = -V_0$$

$$V = V_0 + at$$

$$V + V = at \Rightarrow 2V = at$$

$$V \approx 15$$

$$V_0 \approx 15 \text{ m/s}$$

47.)

$$V_0 = 12.0 \text{ m/s}$$

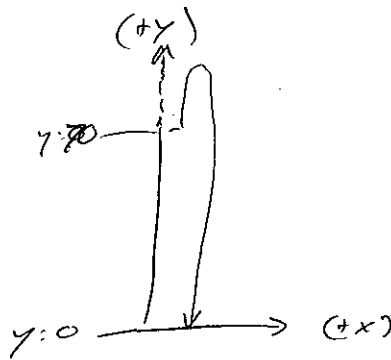
$$V = ?$$

$$y_0 = 70.0 \text{ m}$$

$$t = ?$$

$$y = 0 \text{ m}$$

$$a = -9.8 \text{ m/s}^2$$



a.) $t = ?$

$$y = y_0 + V_0 t + \frac{1}{2} a t^2$$

$$t = \frac{-V_0 \pm \sqrt{V_0^2 - 4(\frac{1}{2}a)y_0}}{2(\frac{1}{2}a)}$$

$$t = \cancel{0.72 \text{ s}} \quad 5.198 \text{ s}$$

b.) $v = ?$

$$V = V_0 + at = 12 \text{ m/s} + (-9.8 \text{ m/s}^2)(5.20 \text{ s}) = -38 \text{ m/s}$$

$$\text{Speed: } 38 \text{ m/s}$$

c.) $V_0 = 12 \text{ m/s}$

$$V = \cancel{12} 0 \text{ m/s}$$

$$a = -9.8 \text{ m/s}^2$$

$$y = ?$$

$$y_0 = 70$$

$$t = ?$$

$$0 = V_0^2 + 2a(y - y_0)$$

$$V_0^2 + 2a(y - y_0) = 0$$

$$y - y_0 = \frac{-V_0^2}{2a}$$

$$y = \frac{-V_0^2}{2a} + y_0 = 77.3469$$

$$\text{Total Distance: } 2(77.3469 - 70) + 70 = \boxed{84.69 \text{ m}}$$

Ch. 3: Extra:

10.) $\vec{D} = \vec{A} + \vec{B} + \vec{C}$

$$D_x = A_x + B_x + C_x$$

$$D_y = A_y + B_y + C_y$$

$$\theta_A = 28.0^\circ \text{ from } x$$

$$\theta_B = 56.0^\circ \text{ from } -x \text{ OR } (180 - 56.0^\circ) \text{ from } x$$

$$\theta_C = -90^\circ \text{ from } x$$

$$D = \sqrt{D_x^2 + D_y^2}$$

$$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right)$$

11.) $\vec{A} - \vec{C} = \vec{A} + (-\vec{C})$

~~Circle~~

$$(\vec{A} - \vec{C})_x = A_x + (-C_x)$$

$$(\vec{A} - \vec{C})_y = A_y + (-C_y)$$

$$|\vec{A} - \vec{C}| = \sqrt{(A_x - C_x)^2 + (A_y - C_y)^2}$$

12.) a.) $\vec{B} - \vec{A}$

$$B_{Ax} = B_x + -A_x$$

$$B_{Ay} = B_y + -A_y$$

b.) $\vec{A} - \vec{B}$

$$A_{Bx} = A_x + -B_x$$

$$A_{By} = A_y + -B_y$$

Equal But Opposite.

Projectile Motion: 2-D Kinematics

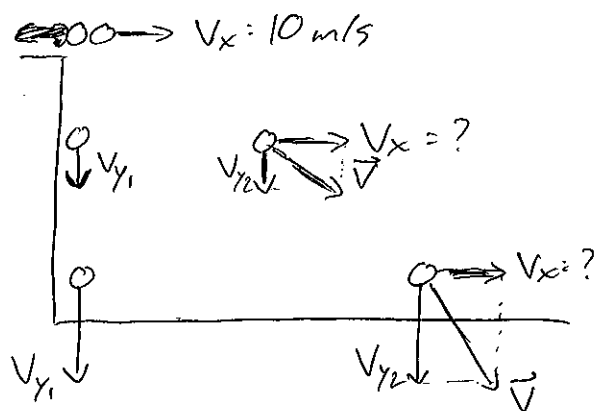
Similar to 1-D Kinematics... just twice the work. with some vectors thrown in

Recall:

$$V = V_0 + at \quad X = X_0 + V_0 t + \frac{1}{2} a t^2$$

$$V^2 = V_0^2 + 2a(x - x_0) \quad \bar{V} = \frac{V + V_0}{2}$$

Conceptual: If I shoot a cannon ball off a cliff, and I drop one at the same time, which hits the ground first (~~assuming~~ ignoring air resistance)?

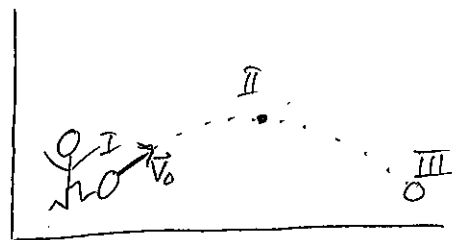


Why?

Where ~~total~~ would the shot cannon's velocity vector point right as it hits the ground?

Note: Vertical acceleration does not affect horizontal motion!
Horizontal acceleration does not affect vertical motion!

What if I kick a ball?



$$\vec{V}_0 = 100 \text{ m/s } @ 40^\circ$$

I. What is V_{0x} ? ($V_{0x} = V_0 \cos \theta$)

What is V_{0y} ? ($V_{0y} = V_0 \sin \theta$)

II. What is V_x ? ($V_x = V_{0x}$)

What is V_y ? (0)

III. What is V_x ? ($V_x = V_{0x}$)

What is V_y ? ($V_y = -V_{0y}$)

What is $\vec{V}$? ($\vec{V} = 100 \text{ m/s } @ -40^\circ$)

2-D Kinematic Equations: We break these problems into components
See pg. 56!!

$$V_x = V_{x0} + a_x t$$

$$x = x_0 + V_{x0} t + \frac{1}{2} a_x t^2$$

$$V_x^2 = V_{x0}^2 + 2a_x(x - x_0)$$

$$V_y = V_{y0} + a_y t$$

$$y = y_0 + V_{y0} t + \frac{1}{2} a_y t^2$$

$$V_y^2 = V_{y0}^2 + 2a_y(y - y_0)$$

Projectile Motion:

Horiz. $V_x = V_{x0}$

$a_x = 0$ $x = x_0 + V_{x0} t$

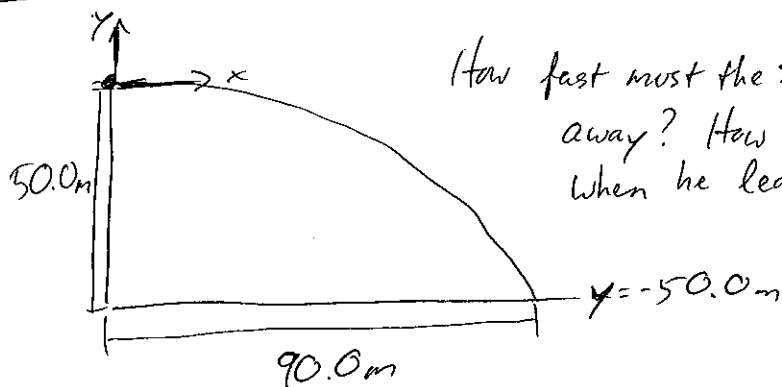
$V_x = \text{const}$

Vert.: $V_y = V_{y0} - g t$

$a_y = -g$ $y = y_0 + V_{y0} t - \frac{1}{2} g t^2$

$$V_y^2 = V_{y0}^2 - 2g(y - y_0)$$

Example 3-4: pg. 57



How fast must the stunt driver go to land 90.0m away? How fast must he ~~be~~ be traveling when he leaves the cliff?

$x_0 = 0$ $y_0 = 0$

$x = 90.0\text{m}$ $y = -50.0\text{m}$

$a_x = 0$ $a_y = -g = -9.80\text{m/s}^2$

$V_{x0} = ?$ $V_{y0} = 0$

$V_x = ?$ $V_y = ?$

$t = ?$

Example: cont...

$$y = \cancel{y_0} + \cancel{v_{y0}}t + \frac{1}{2}a_y t^2$$

$$y = \frac{1}{2}a_y t^2 \Rightarrow t^2 = \frac{2y}{a_y}$$

$$t = \sqrt{\frac{2y}{a_y}} = 3.19s$$

$$x = \cancel{x_0} + \cancel{v_{x0}}t + \frac{1}{2}a_x t^2$$

$$x = v_{x0}t \Rightarrow v_{x0} = \frac{x}{t} = 28.2 \text{ m/s}$$

Questions: ~~Riding in car. Have tennis ball. Throw it straight up.~~

~~What happens?~~ Q. 15

- Boy hanging from tree. Another boy with water balloon slingshot aimed directly at tree boy. When launched, tree boy lets go. What happens?

Q. 20

Problems: ~~Q, 22, 24~~

20, 27, 24

Extra: 25, 28, 33

For HW: 27, 24

25, 28, 33

25. Determine how much farther a person can jump on the moon if V_0 and θ are same.

$$g_{\text{moon}} = \frac{1}{6}g$$

$$X = X_0 + V_{0x}t + \frac{1}{2}a_x t^2$$

$$X = X_0 + V_0 \cos \theta t$$

$$V_y = V_{y0} + g_{\text{moon}} t$$

$$-2V_0 \sin \theta = g_{\text{moon}} t \Rightarrow \boxed{t = \frac{-2V_0 \sin \theta}{g}}$$

$$X = X_0 + V_0 \cos \theta \left(\frac{-2V_0 \sin \theta}{g} \right)$$

$$X = \frac{-2V_0^2 \cos \theta \sin \theta}{g} \quad \text{On Earth}$$

$$X = 6 \left(\frac{-2V_0^2 \cos \theta \sin \theta}{g} \right) \quad \text{On Moon}$$

6 times!

28.1 Show that $V_0 = V_f$

$$V_x = V_{x0} + a_x t$$

$$V_x = V_{x0}$$

$$V_y^2 = V_{y0}^2 + 2a_y(y - y_0)$$

Note: $y = y_0$

$$V_y^2 = V_{y0}^2$$

$$V_0^2 = V_{y0}^2 + V_{x0}^2 = V_y^2 + V_x^2 = V^2$$

33.1 At what projection angle will range equal max height.

At what θ will ~~the~~ $x = \frac{1}{2}y$?

$$x = x_0 + V_{x0}t + \frac{1}{2}a_x t^2$$

$$x = V_0 \cos \theta t \Rightarrow t = \frac{x}{V_0 \cos \theta}$$

$$y = y_0 + V_{y0}t + \frac{1}{2}a_y t^2$$

$$y = 2x = \frac{V_0 \sin \theta}{V_0 \cos \theta} x + \frac{1}{2}(-g) \frac{x^2}{V_0^2 \cos^2 \theta}$$

$$V_y^2 = V_{y0}^2 - 2g(y - y_0)$$

$$2gy = V_{y0}^2$$

$$4gx = V_0^2 \sin^2 \theta = V_0^2 (1 - \cos^2 \theta)$$

$$x \cdot V_0 \cos \theta = \frac{1}{2} (V_0 \sin \theta - \frac{1}{2} g t^2)$$

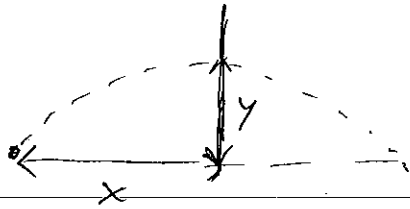
$$V_y = V_{y0} + a_y t$$

$$V_0 \sin \theta = gt$$

$$V_0 \cos \theta = \frac{1}{2} (V_0 \sin \theta - \frac{1}{2} V_0 \sin \theta)$$

$$\cos \theta = \frac{1}{4} \sin \theta \Rightarrow 4 = \tan \theta \Rightarrow \theta = 76^\circ$$

33. At what θ will $2x = y$



$$x = \cancel{V_0} + V_{x0}t + \frac{1}{2}a_x t^2$$

$$x = V_{x0}t = V_0 \cos \theta t$$

$$y = \cancel{V_0} + V_{y0}t + \frac{1}{2}a_y t^2$$

$$y = V_0 \sin \theta t + \frac{1}{2}g t^2$$

$$\cancel{V_y} = V_{y0} + a_y t$$

$$0 = V_{y0} - g t \Rightarrow g t = V_{y0} = V_0 \sin \theta$$

$$y = V_0 \sin \theta t - \frac{1}{2} V_0 \sin \theta t = \frac{1}{2} V_0 \sin \theta t$$

$$\text{if } x = \frac{1}{2} y$$

$$\cancel{V_0} \cos \theta t = \frac{1}{2} \left(\frac{1}{2} V_0 \sin \theta t \right) = \frac{1}{4} \cancel{V_0} \sin \theta t$$

$$4 = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\boxed{\theta = \tan^{-1}(4) = 76^\circ}$$

35. $y_0 = 235 \text{ m}$

$v_x = 69.4 \text{ m/s}$

a.) $y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$

$y_0 = \frac{1}{2}gt^2 \Rightarrow t^2 = \frac{2y_0}{g}$

$t = \sqrt{\frac{2y_0}{g}}$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$x = v_{x0} \sqrt{\frac{2y_0}{g}} = 481 \text{ m}$

b.) $x_0 = 0$ $x = 425 \text{ m}$

$y_0 = 235 \text{ m}$

$v_{x0} = 69.4 \text{ m/s}$

$y = 0$

$v_{y0} = ?$

$a_y = -9.8 \text{ m/s}^2$

$v_y = ?$

$v_y = v_{y0} - gt$ $y - y_0 = v_{y0}t - \frac{1}{2}gt^2$

$v_{y0} = v_y + gt \Rightarrow y - y_0 = v_y t + gt^2 - \frac{1}{2}gt^2$

$0 - y_0 = v_y t + \frac{1}{2}gt^2$

$-y_0 = v_y t + \frac{1}{2}gt^2$

$-\frac{y_0}{t} - \frac{1}{2}gt = v_y$

$x = x_0 + v_{x0}t$

$t = \frac{x}{v_{x0}}$

$v_y = -\frac{v_{x0}y_0}{x} - \frac{1}{2}g\frac{x}{v_{x0}} = -68.38 \text{ m/s}$

$v_{oy} = v_y + gt = v_y + g\frac{x}{v_{x0}} = -8.4 \text{ m/s}$

20.1 $y = 4.5 \text{ m}$ $V = ? = V_x$
 $x = 5.0 \text{ m}$ $V_y = 0$
 $x_0 = 0 \text{ m}$ $t = ?$
 $y_0 = 0 \text{ m}$

$$V_y^2 = V_{y0}^2 - 2g(y - y_0)$$

$$V_{y0}^2 = 2gy \Rightarrow V_{y0} = \sqrt{2gy} = 9.39149$$

~~$$y = y_0 + V_{y0}t + \frac{1}{2}gt^2$$~~

~~$$\frac{1}{2}gt^2 = \sqrt{2gy}t + y > 0$$~~

~~$$a = \frac{1}{2}(+9.8) \quad b = 9.39 \quad c = 4.5$$

 $= 4.9$~~

~~$$t = \frac{\sqrt{2gy} \pm \sqrt{2gy - 2gy}}{2(\frac{1}{2}g)} = \frac{\sqrt{2gy}}{g} = 0.958315 \text{ s}$$~~

$$V_y = V_{y0} + a_y t \Rightarrow a_y t = -V_{y0} = -\sqrt{2gy}$$

$$\boxed{t = \frac{-\sqrt{2gy}}{-g} = 0.958315 \text{ s}}$$

$$V_x = \frac{x}{t} = \frac{5.0}{0.958315}$$

27.)

$$V_{0x} = 180 \text{ km/h}$$

$$\vec{v} \rightarrow V_{0x} = 180 \text{ km/h}$$

$$y_0 = 160 \text{ m} = .160 \text{ km} \quad \text{"What is } t?"$$

$$V_y = V_{y0} + a_y t \Rightarrow t = \frac{V_y}{a_y}$$

Finding V_y :

$$V_y^2 = V_{y0}^2 + 2a_y(y - y_0)$$

$$V_y^2 = -2a_y y_0 \Rightarrow V_y = \sqrt{-2a_y y_0}$$

$$t = \frac{\sqrt{-2a_y y_0}}{a_y} = 5.71 \text{ s}$$

24.)

$$\theta = 28.0^\circ$$

$$V_0 = ?$$

$$x = 7.80 \text{ m}$$

$$V_{0x} = V_x$$

$$x_0 = 0$$

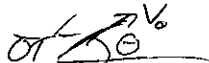
$$y = 0$$

$$y_0 = 0$$

$$V_{0y} = V_0 \sin \theta = -V_y$$

$$a_y = -9.8 \text{ m/s}^2$$

$$t = ? = \frac{2V_0 \sin \theta}{a_y}$$



$$V_y = V_{y0} + a_y t$$

$$2V_{0y} = a_y t \Rightarrow t = \frac{2V_{0y}}{a_y}$$

$$x = x_0 + V_0 \cos \theta \left(\frac{2V_0 \sin \theta}{a_y} \right) + 0$$

$$x = \frac{V_0^2 \cos \theta \sin \theta}{a_y}$$

b.) What is x if $V_0 \rightarrow 1.05 V_0$.

19, 27, 29

19.) $V_0 = 6.8 \text{ m/s}$ $V_{0x} = V_0 \cos \theta$ $V_x = V_{0x}$

$x = 1.0 \text{ m}$

$V_{0y} = V_0 \sin \theta$

$x_0 = 0$

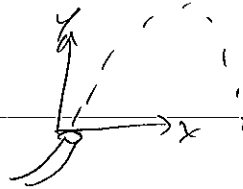
$a_y = -9.8 \text{ m/s}^2$

$y_0 = 0$

$a_x = 0 \text{ m/s}^2$

$y = ?$

$t = ?$



$V_x^2 = V_{0x}^2 + 2a_x(x - x_0)$ No good.

Find t:

~~$y = y_0 + V_{y0}t + \frac{1}{2}a_y t^2$~~

~~$V_y = V_{y0} + a_y t$~~

~~$0 = V_{y0} + \frac{1}{2}a_y t^2$~~

$y = y_0 + V_{y0}t + \frac{1}{2}a_y t^2$

~~$V_{y0} = -\frac{1}{2}a_y t^2$~~

$0 = V_{y0}t + \frac{1}{2}a_y t^2$

$-V_{y0}t = \frac{1}{2}a_y t^2$

$t = \frac{-2V_{y0}}{a_y} = \frac{-2V_0 \sin \theta}{a_y}$

~~$V_{y0} = -\frac{1}{2}a_y t^2$~~

$x = x_0 + V_{x0}t = V_0 \cos \theta$

19.) $V_0 = 6.8 \text{ m/s}$ $a_y = -9.8 \text{ m/s}^2$

$x = 1.0 \text{ m}$ $V_{0y} = V_0 \sin \theta$

$x_0 = 0$ $V_y = 0$

$y_0 = 0$ $V_{0x} = V_x = V_0 \cos \theta$

$y = ?$ $t = ?$

$V_y = V_{y0} + a_y t$

$V_y^2 = V_{y0}^2 + 2a_y (y - y_0)$

$t = \frac{-V_{y0}}{a_y}$

$V_{y0}^2 = 2a_y y$

$y = \frac{V_{y0}^2}{2a_y}$

$y = y_0 + V_{y0} t + \frac{1}{2} a_y t^2$

$y = V_{y0} \cdot \left(-\frac{V_{y0}}{a_y}\right) + \frac{1}{2} a_y \frac{V_{y0}^2}{a_y^2} = -\frac{V_{y0}^2}{a_y} + \frac{1}{2} \frac{V_{y0}^2}{a_y} = -\frac{V_{y0}^2}{2a_y}$

$y = \frac{-V_{y0}^2}{2a_y} = \frac{2a_y}{V_{y0}^2} \Rightarrow -V_{y0}^4 = 4a_y^2$

$V_{y0} = (-4a_y^2)^{1/4}$